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An analysis of the cost of warehousing logistics based on the important nodes of complex networks

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Abstract. The optimization of logistic distribution route is a sort of NP-complete problem with high practical value. Given the disadvantages of low search rate and sensitivity to locally optimal solution in the traditional heuristic optimization algorithm, a method for the optimization of logistic distribution route of quantum ant colony algorithm (QACA) is presented. Firstly, a mathematical model is established based on the optimization problem of logistic distribution route. Secondly, solutions are obtained with QACA. The pheromone on the routes is subject to quantum bit encoding. The quantum rotation gate and the optimal route are used to update the pheromone. Lastly, QACA performance receives the stimulation test the results of which show that QACA has a higher capacity of global search and higher convergence rate, which may effectively solve the problem of logistic distribution route.

Key words. Quantum computation, Cost, Pheromone, Logistic distribution, Route selection
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1. Introduction

The problem of logistic distribution route optimization is a key part of logistic distribution. An appropriate arrangement for the number of vehicles and vehicle rout is an important tool to reduce wastes and improve economic benefits, which has significant influence on the speed, cost and benefits throughout logistic distribution.

The problem of logistic distribution route optimization is a typical combinatorial optimization problem. This is a non-deterministic polynomial complete (NPC) problem. It is hard for traditional manual arrangement for the distribution route to satisfy the business requirements of modern enterprises. Application of computers to route arrangement must be imperative [2]. There are many ways to solving

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the problem of distribution route optimization, all of which may be grouped into 2 categories: accurate algorithm and heuristic algorithm. The former consists of enumeration and dynamic programming methods [3, 4]. Such methods offer larger amount of computation and higher storage capacity, which make them applicable to the solutions to the optimization problem of small-scale logistic distribution route. However, the latte can obtain solutions with better quality with shorter time, such as genetic algorithm, stimulated annealing algorithm, particle swarm optimization, ant colony algorithm all of which are for the optimization of logistic distribution route [5-8]. The ant colony algorithm (ACA) is featured with higher optimization capacity, robustness and good distributed computing. ACA is mostly used in the application of logistic distribution route optimization, making it a major research direction. However, ACA does have some deficiencies, such as low solution rate and sensitivity to locally optimal solution [9]. The quantum ant colony algorithm (QACA) combines quantum computation and ACA and introduce the state vector and quantum rotation gate in quantum computation to ACA, making the convergence rate of the algorithm higher. This algorithm has been successfully applied to the multi-objective combinatorial optimization problem including TSP solution, image coloring and function optimization [10].

To obtain a better logistic distribution route optimization scheme, a method for the optimization of logistic distribution route of quantum ant colony algorithm (QACA) is presented. A mathematical model for the optimization of logistic distribution route is first established. QACA algorithm is then used to solve it. The last step is that a stimulation experiment is carried out to test the effectiveness and superiority of the methods in this paper.

2. Optimization of logistic distribution route and mathematical model

2.1. Description of distribution route problem

There are M customer points in one logistic distribution network. Assuming that the demand qi and location of each customer point $i$is known, at most K vehicles will be needed for delivery from the distribution center to the demand point. Each vehicle leaves from and finally returns to the distribution center. If the maximum loading capacity of each vehicle k is fixed, it will be required that the arrangement for the traveling route of the vehicle shall make the minimal total cost (such as distance, time, etc.) subject to satisfaction of the following constraint conditions:

(1) The location of the distribution center is known and unique;

(2) The sum of the demand of customer points on each line does not exceed the vehicle loading capacity;

(3) The total length of each distribution route does not exceed the maximum traveling distance of one delivery of the vehicle; and

(4) The demand of each customer point must and can only be completed by one vehicle.

2.2. Mathematical model of logistic distribution route optimization

Assuming that the distance for the customer to travel from point i to point j is $b_{i,j}$, and $i, j = 0, 1, \cdots, M$, and $b_{0,0}$ denotes the distribution center, the mathematical model of logistic distribution route optimization will be:

$$MinF = \sum_{k=1}^{K} \left(\sum_{i=1}^{n_k} b_{r_k^{i-1}, r_k^i} + b_{r_k^{nk}, 0} \right) \times \operatorname{sgn}(n_k) \tag{1}$$

Where, n_k denotes the total number of customers for delivery by vehicle k. When $n_k = 0$, it represents that vehicle k is not involved in the distribution, say:

$$\operatorname{sgn}(n_k) = \begin{cases} 1 & n_k = 0 \\ 0 & n_k \geq 1 \end{cases} \tag{2}$$

The constraint condition for the optimization of logistic distribution route is:

$$\begin{cases} \sum\limits_{i=1}^{n_k} p_{r_k^i} \leq pk; n_k \neq 0 \\ \sum\limits_{i=1}^{n_k} b_{r_k^{i-1}, r_k^i} + b_{r_k^{n_k}, 0} \leq B_k; n_k \neq 0 \\ R_{k1} \cap R_{k2} = \phi, k_1 \neq k_2 \\ \bigcup\limits_{k=1}^{K} R_k = \{1, 2, \cdots, M\}; 0 \leq n_k \leq M \end{cases} \tag{3}$$

Where, B_k denotes the maximum traveling distance of vehicle k; R_k denotes the set of customer points for delivery by vehicle k; r_k^j denotes that the sequence of the customer in the distribution route of vehicle k is j.

According to Formula (1), logistic distribution requires not only the least number of vehicles for distribution but also the shortest distribution route. In addition, it requires the delivery of the goods to the customer within the given time limit. It is to find an optimal logistic distribution route which satisfies multiple constraint conditions [11].

3. Quantum ant colony algorithm of logistic distribution route optimization

Inspired by quantum-inspired evolutionary algorithm (QEA), Li Panchi et al. proposed the quantum ant colony algorithm (QACA) by integrating quantum computation and ant colony algorithm [12]. What comes first in this method is that each ant carries a group of quantum bit representing the information about current location of the ant. The advancing objective of the ant is selected based on the intensity of pheromone and the selective probability of visibility structure. The next step is that the quantum rotation gate is used to update the quantum bit the ant carries; the quantum non-gate is used to achieve the variation of the location where the ant is and increase diversity of the location. The last step is that the intensity of pheromone and visibility of the ant colony is updated according to the location after movement. All these may better solve the problem of low convergence rate and sensitivity to locally optimal solution in ACA algorithm during the course of solving.

3.1. Quantum information encoding

Quantum bit (qubit) is used to represent information in quantum computation. A simple qubit is a two-state system. If a qubit is expressed in probability amplitude $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, the individual probability amplitude of n qubit(s) will be expressed in:

$$\left[\begin{array}{c|c|c|c} \alpha_1 & \alpha_2 & \dots & \alpha_n \\ \beta_1 & \beta_2 & \dots & \beta_n \end{array}\right]. \tag{4}$$

Where, the quantum individual may represent the superposition state of any quantum when α_i and β_i satisfies that $|\alpha_i|^2 + |\beta_i|^2 = 1$, $i = 1, 2, \dots, n$.

In QACA, qubit is used to represent the pheromone on the route. The quantum information encoding of ant k may be expressed in:

$$Q\tau_k = \begin{pmatrix} \begin{pmatrix} \alpha_{11} \\ \beta_{11} \end{pmatrix} & \begin{pmatrix} \alpha_{12} \\ \beta_{12} \end{pmatrix} & \cdots & \begin{pmatrix} \alpha_{1n} \\ \beta_{1n} \end{pmatrix} \\ \begin{pmatrix} \alpha_{21} \\ \beta_{21} \end{pmatrix} & \begin{pmatrix} \alpha_{22} \\ \beta_{22} \end{pmatrix} & \cdots & \begin{pmatrix} \alpha_{2n} \\ \beta_{2n} \end{pmatrix} \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} \alpha_{n1} \\ \beta_{n1} \end{pmatrix} & \begin{pmatrix} \alpha_{n2} \\ \beta_{n2} \end{pmatrix} & \cdots & \begin{pmatrix} \alpha_{nn} \\ \beta_{nn} \end{pmatrix} \end{pmatrix}. \tag{5}$$

Where, n is the number of customer(s). $\begin{pmatrix} \alpha_{ij} \\ \beta_{ij} \end{pmatrix}$ denotes the probability amplitude of the pheromone on the distribution route between customer i and customer j, and there is:

$$\begin{cases} |\alpha_{ij}|^2 + |\beta_{ij}|^2 = 1, & if\ \ i \neq j \\ |\alpha_{ij}|^2 = |\beta_{ij}|^2 = 0, & if\ \ i = j \end{cases} \tag{6}$$

For customers i and j, when an ant passes by the route from customers $i$to j, the probability amplitude β_{ij} of the pheromone on the route distance will increase, and the pheromone will be stronger, or the pheromone on the route will volatilize.

3.2. Rules of pheromone updating

When all ants establish the route, the pheromone on each route length will be updated. Firstly, the pheromone on all sides will decrease by the size of a constant factor. The pheromone on the route by which these ants pass will increase. The evaporation of pheromone is executed within the formula below:

$$\tau_{ij} = (1-\rho)\,\tau_{ij}, \forall\,(i,j) \in E. \tag{7}$$

Where, ρ is the evaporation rate of pheromone, and $0 < \rho \leq 1$. Parameter ρ plays a role in avoiding the unlimited accumulation of pheromone. Following the evaporation step of pheromone, all ants will release pheromone on the route by which they pass:

$$\tau_{ij} = \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}, \forall\,(i,j) \in E. \tag{8}$$

Where, $\Delta\tau_{ij}^{k}$ is the amount of pheromone releases by ant k to the distribution route it passes by.

3.3. Adjustment of quantum rotation gate

Assuming that there is (are) m ant(s). $n \times n$ matrix R is a solution route from the distribution center to all customers solved for n customer logistic system(s). $R[i,\ j]$ $= 1$ denotes that there is a side from customers i to j in route R. When $i = j$, there must be $R[\mathrm{i},\ \mathrm{j}] = 0$. Matrix $R\mathrm{k}$ ($k = 1, 2, \ldots, m$) is used in the algorithm to record the route obtained by ant k. R best records the optimal solution obtained in the operational process. The quantum probability amplitude of the ant on each route is updated by using the quantum rotation gate. The way to adjust the quantum rotation gate is:

$$\begin{pmatrix} \alpha_{ij}^{t+1} \\ \beta_{ij}^{t+1} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha_{ij}^{t} \\ \beta_{ij}^{t} \end{pmatrix}. \tag{9}$$

Where, $\left(\alpha_{ij}^{t}, \beta_{ij}^{t}\right)^{T}$ is the probability amplitude of pheromone on the route between customers i and j in iteration t; θ denotes the rotation angle of the qubit from routes i to j.

3.4. Steps to solving logistic distribution route optimization

Step1: by setting the values of parameters α, β, ρ and γ, the number of ant is m, the maximum number of iteration is N MAX, the number of current iteration is $t = 0$, and the pheromone is $\tau_{ij}(0) = 1$. To ensure simultaneous occurrence of all states in the same probability in the initial search of the algorithm, the values of all α_{ij}, β_{ij} in the quantum pheromone encoding of the ant will be $1/\sqrt{2}$.

Step2: Assuming that m ant(s) is (are) brought to the logistic distribution center, and each ant individually builds up a solution, the next customer is selected with

Formula (10) according to the constraint condition of logistic distribution in Formula (3). The state transition rule will be repeatedly applied until ant k completes the logistic distribution of all customers.

$$j = \begin{cases} \arg\max\limits_{l \in N_i^k}\{\tau_{il}(\eta_{il})^{\delta}\}, & if \ \ q \le q_0 \\ J \qquad\qquad\qquad\quad , & otherwise \end{cases} \tag{10}$$

Where, τ_{il} is the pheromone concentration on distribution route $(i,\, l)$; η_{il}=$1/C_{il}$ represents the self-heuristics quantity of distribution $(i,\, l)$; δ is the weight of self-heuristics quantity; N_i^k represents the set of adjacent customers to which ant k on customer i may arrive directly. It also means the set of all customers who have not been visited by k.

Customer j is a random variable generated in a roulette way with the probability distribution given in Formula (10).

$$p_{ij}^k = \frac{[\tau_{ij}]^{\alpha}\,[\eta_{ij}]^{\beta}}{\sum\limits_{l \in N_i^k} [\tau_{il}]^{\alpha}\,[\eta_{il}]^{\beta}}, \quad if \ j \in N_i^k. \tag{11}$$

Where, parameters α and β determine the relative influence of pheromone and heuristic information, respectively; p_{ij}^k refers to the probability with which ant k on customer i selects customer j as the next customer to be visited.

Step3: if m ant(s) has (have) built up their own solution, we may go to Step4; otherwise, we will go to Step2.

Step4: with current optimal solution, the quantum rotation gate is applied to update the quantum information probability amplitude of the ant on each distribution route. The pheromone can be updated with Formulas (7) and (8).

Step5: if the end condition is met, i.e. $t > N\max$, the optimal solution will be exported to obtain the optimal scheme for logistic distribution route. If not, $t = t{+}1$, we will go to Step2 for continued performance.

4. Stimulation experiment

4.1. Classical function test

3 classical standard multi-peak functions are selected for the test and experiment. The test results are compared with ACA. These three classical test functions are as follows:

(1) Rastrigin function

$$f(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10),. \tag{12}$$

(2) Ackley function

$$f(x) = -20\exp\left(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{30}\sum_{i=1}^{n}\cos 2\pi x_i\right) + 20 + e \tag{13}$$

(3) Schaffer function

$$f(x) = \frac{\sin^2\sqrt{x_1^2 + x_2^2} - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2} - 0.5 \tag{14}$$

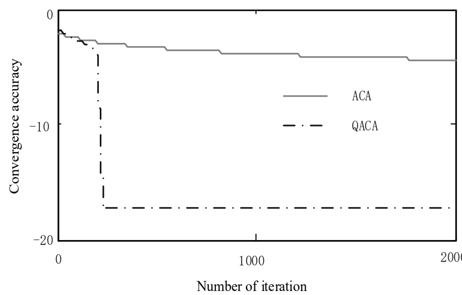
Fig. 1 is the logarithmic evolution curve of the fitness of 3 test functions (note: to facilitate the display and observation of the evolution curve, the logarithm with 10 used as the base is selected for the function fitness value. According to Fig. 1, QACA will soon arrive at the theoretical minimum points 0 and -1 for all functions. The convergence rate of QACA is evidently better than that of ACA algorithm mainly due to the fact that QACA uses quantum bit for pheromone encoding. The quantum rotation gate updates the pheromone in the link to avoid early occurrence of stagnation and trapping in locally optimal solution as drawbacks of ACA.

4.2. Stimulation test of logistic distribution route optimization

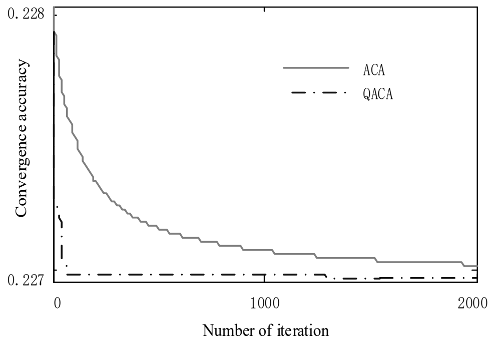
A company has a logistic distribution center comprising 5 cargo carrier (each has a loading capacity of 1 ton). Delivery is required to be made to 7 customer points. The coordinates and demand for goods of each customer point are shown in Table 1 (0 denotes the distribution center; 1-7 means the customer points).

Table 1. Customer coordinates and demand of goods

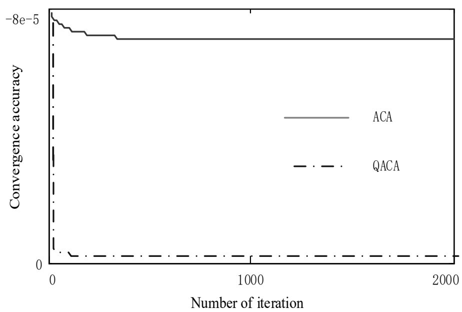
Customer No.	Coordinates	Demand of goods
0	(40, 40) (40, 40)	
1	(10, 20) (10, 20)	1
2	(15, 50) (15, 50)	1.6
3	(25, 40) (25, 40)	1.3
4	(30, 60) (30, 60)	2.4
5	(35, 15) (35, 15)	1.5
6	(55, 45) (55, 45)	1.1
7	(65, 10) (65, 10)	1.6



(a) Convergence curve of rastrigin function fitness



(b) Convergence curve of ackley function fitness



(c) Convergence curve of schaffer function fitness

Fig. 1. Comparison between QACA and ACA algorithms in convergence

For QACA, number of ant $n = 5$, $\alpha = 1, \beta = 5, \rho = 0.9$, $\gamma = 2 \sim 4$, prior knowledge $q_0 = 0.05$, maximum evolution algebra $N\mathrm{MAX} = 500$, the initialization pheromone on each side is 1. ACA and QACA are used to solve the problem of logistic distribution route optimization in Table 1. The results obtained are shown in Fig. 2 and 3.

According to Fig. 2, the logistic distribution route of ACA consists of 2 lines: Line 1: 0→4→2→3→1→0, the total length of the distribution route is 110.547km; Line 2: 0→5→7→6→0, the total length of the distribution route is 108.121km. In this case, the total route length under ACA logistic distribution route scheme will be 218.668km.

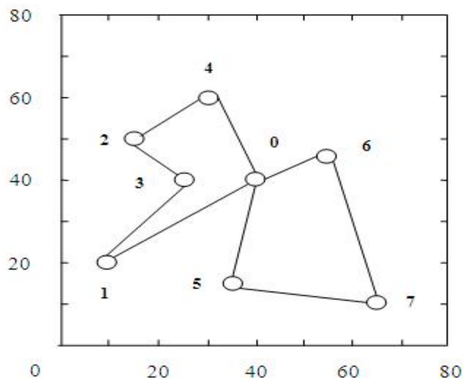


Fig. 2. Logistic distribution route of ACA

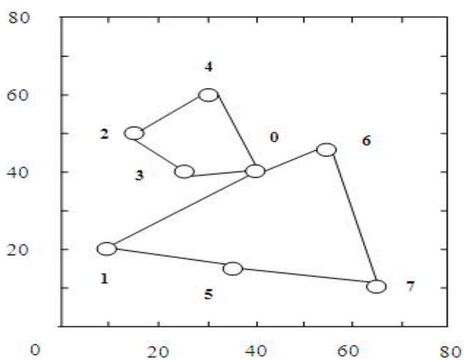


Fig. 3. Logistic distribution route of QACA

According to Fig. 3, the logistic distribution route of QACA consists of 2 lines: Line 1: 0→4→2→3→0, the total length of the distribution route is 64.491km; Line 2: 0→1→5→7→6→0, the total length of the distribution route is 144.177km. In this case, the total route length under QACA logistic distribution route scheme will be 208.668km.

A comparison between the results in Fig. 2 and Fig. 3 shows that QACA will finds a logistic distribution route scheme better than that of ACA. The main reason is that QACA encodes the pheromone on the distribution routes with qubit. The quantum rotation gate updates the pheromone of the distribution route, which improves the optimization capability of the algorithm and effectively avoids sensitivity of the algorithm to locally optimal solution. It also prevents early convergence and enhances the search efficiency.

5. Conclusion

A QACA logistic distribution route optimization is presented for the characteristics of logistic distribution route optimization and the drawbacks in ACA algorithm.

The experiment results show that QACA obtains the optimal solution of the logistic distribution route both quickly and effectively, which is of reference value in the study of ACA algorithm and logistic distribution route problems.

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